Section D

7. (a) If $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$, then prove that $\nabla.\left(r^n \vec{r}\right) = n(n+3)r^n$. **5**

(b) Find the directional derivative of the function $f(x) = x^2 - y^2 + 2z^2$ at the point P (1, 2, 3) in the direction of the line PQ, where Q is the point (5, 0, 4). **5**

8. (a) Using Stoke's theorem, evaluate $\int_C \left((2x - y)\,dx - yz^2 dy - y^2 z\, dz\right)$, where C is the circle $x^2 + y^2 = 1$, corresponding to the surface of the sphere of unit radius. **5**

(b) Use Green's theorem in the plane for $\oint_C \left(3x^2 - 8y^2\right)dx + (4y - 6xy)\,dy$, where C is the boundary of the region defined by $y = \sqrt{x}$, $y = x^2$. **5**

(Compulsory Question)

9. Attempt all parts of this question : **2×10=20**

(a) Sum of eigen values of matrix is ....

Roll No. ....

Total Pages : 05

July-22-00207

B. Tech. EXAMINATION, 2022

Semester I (CBCS)

ENGINEERING MATHEMATICS–I (A & B)

MA-101

Time : 3 Hours *Maximum Marks : 60*

The candidates shall limit their answers precisely within the answer-book (40 pages) issued to them and no supplementary/continuation sheet will be issued.

Note : Attempt *Five* questions in all, selecting *one* question from each Section A, B, C and D. Q. No. 9 is compulsory.

Section A

1. (a) Find the values of a and b for which the equations $x + ay + z = 3$, $x + 2y + 2z = b$, $x + 5y + 3z = 9$ are consistent. When will these equations have a unique solution ? **5**

(b) Find the eigen values and eigen vectors of the matrix $A = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$. **5**

2. Find the characteristic equation of the matrix $A = \begin{pmatrix} 1 & 3 & 7 \\ 4 & 2 & 3 \\ 1 & 2 & 1 \end{pmatrix}$. Show that the equation is satisfied by A and hence obtain the inverse of given matrix. **10**

Section B

3. (a) Prove that $\tan\left(i\log\left(\frac{a-ib}{a+ib}\right)\right) = \frac{2ab}{a^2-b^2}$. **5**

(b) Solve the equation :

$x^6+x^5+x^4+x^3+x^2+x+1=0$ by complex root method. **5**

4. (a) Sum the series to infinity

$$\sin\alpha + x\sin(\alpha+\beta) + \frac{x^2}{2!}\sin(\alpha+2\beta) + \ldots\ldots.$$ **5**

(b) If $\tan(\theta+i\phi) = \cos\alpha + i\sin\alpha = e^{i\alpha}$, prove that :

$$\theta = \frac{n\pi}{2} + \frac{\pi}{4} \text{ and } \phi = \frac{1}{2}\log\tan\left(\frac{\pi}{4}+\frac{\alpha}{2}\right).$$ **5**

Section C

5. (a) Examine for maximum and minimum values of $\sin x + \sin y + \sin(x+y)$. **5**

(b) If u is a homogenous function of degree n in x and y, then prove that :

$$x^2\frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 u}{\partial x\partial y} + y^2\frac{\partial^2 u}{\partial y^2} = n(n-1)u.$$ **5**

6. (a) Define Gamma function. Also find the value of $\Gamma\left(\frac{1}{2}\right)$. **5**

(b) Evaluate $\iint_R e^{2x+3y}\,dxdy$ over the triangle bounded by $x=0,\ y=0$ *and* $x+y=1$. **5**

(b) Prove that the inverse of an orthogonal matrix is orthogonal.

(c) Define rank of a matrix with one example.

(d) Find the general value of $\log(1+i)$, where $i=\sqrt{-1}$.

(e) Separate real and imaginary parts of $\cos(x+iy)$.

(f) If $\vec{r}=x\hat{i}+y\hat{j}+z\hat{k}$, then prove that $div\,\vec{r}=3$.

(g) Define homogenous function. Write the statement of Euler's theorem on homogenous function of first order.

(h) Find first order partial derivative of $u=\tan^{-1}\left(\dfrac{x^2+y^2}{x+y}\right)$.

(i) If $\vec{r}=x\hat{i}+y\hat{j}+z\hat{k}$, then show that $\text{grad}\left(\dfrac{1}{r}\right)=-\dfrac{\vec{r}}{r^3}$.

(j) Evaluate $\iint_S \vec{r}.\hat{n}\,ds$, where S is a closed surface.